INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2017-18
Statistics - III, Midterm Examination, September 14, 2017

1. Consider the model $\mathbf{Y} = X\beta + \epsilon$, where $X_{n\times p}$ has rank $r \leq p$; also $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$. Let $(X'X)^-$ be a generalized inverse of $X'X$, and $\hat{\beta}$ be a least squares estimator of β. Suppose $A\beta$ is estimable where $A_{q\times p}$ has rank q.
(a) Show that $A(X'X)^-X'X(X'X)^-A' = A(X'X)^-A'$.
(b) Find the probability distribution of $A\hat{\beta}$.
(c) Find $E(\hat{\beta}'A'(A(X'X)^-A')^{-1}A\hat{\beta})$. [15]

2. What is meant by Analysis of Variance? Explain its role in multiple regression. [5]

3. Consider the model $\mathbf{Y} = X\beta + \epsilon$, where $X_{n\times p}$ has $\mathbf{1}$ as its first column ($\mathbf{X}_0$) and has rank $r \leq p$; also $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$. Consider testing the usefulness of the predictors, $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{p-1}$, which appear as the last $p-1$ columns of X. Show that the ANOVA F-test is equivalent to GLRT, the generalized likelihood ratio test. [15]

4. Suppose $\mathbf{X} \sim N_k(\mu, \Sigma)$ where $\text{Rank}(\Sigma) = r \leq k$ and let $B_{k\times k}$ be any symmetric matrix such that $B\mu = \mathbf{0}$. Show that $\mathbf{X}'B\mathbf{X}$ has a χ^2 distribution if and only if

$$\Sigma B \Sigma B \Sigma = \Sigma B \Sigma.$$

Find the degrees of freedom of this χ^2 distribution. [15]